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Instability of accelerating aerosol surface

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Abstract

The linear instability of the interphase surface is investigated in present paper for the case when this accelerating surface separates monodisperse suspension of solid particles and homogeneous incompressible fluid. It is found that the solutions of equations of two-phase dynamics have three types of disturbances, in contrary to the case of homogeneous medium. The dispersion relation of the boundary-value problem for perturbations yields the aperiodic unstable root which is inherent only to the behavior of the accelerating heterogeneous medium since it disappears when the dispersive phase density disappears. This root exists simultaneously with the classic Rayleigh–Taylor root. When either the carrying phase viscosity or acceleration increase, or the particle size either wavenumber of disturbance decrease the action of mechanism of classic Rayleigh–Taylor instability becomes dominant. The found type of instability is caused by action of the inertial mass force since it disappears when the acceleration vanishes. The interphase friction is the natural stabilizing mechanism. The found mechanism can influence the dust motion in high atmosphere leading up to the dynamic coagulation of aerosol.

Keywords: Aerosol dynamics; mixing and fluctuations in aerosol transport.

1. Introduction

The problem sometimes arises in the studying of the disperse mixture motions, which is connected with a character of heterogeneous medium interaction with another homogeneous medium when the conditions exist for the development of both types of hydrodynamic instability (Kelvin – Helmholtz and Rayleigh – Taylor) at the surface of separation. The mechanism of instability works intensively in the process of powder dispersion by the explosion energy (Fig. 1; Frost et al., 2011); it can influence the dust motion in high atmosphere leading up to the dynamic coagulation of aerosol (Kotyusov &

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Fig. 1. Explosive powder dispersion (Frost et al., 2011).

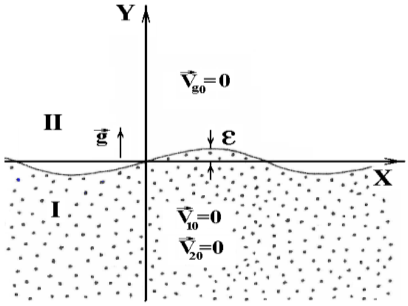

Fig. 2. Disturbed mechanical system.

Niemtzov, 1990; Hinds et alii, 2000); as well, it can fasten the generation of inflammable mixture behind the detonation wave front in aerosol (Aslanov & Girin, 1988).

In order to determine the influence of heterogeneity it is insufficient to use the models of homogeneous fluid mechanics and the enlisting of two-velocity models is required. The stability of solutions of equations of such models has been widely investigated recently at studying of liquid motion in pipelines (Rudyak & Isakov, 1996), including applications to the appearance of jam-motions at hydrocarbons raw materials transportation (Bernicot & Deheuvels, 1995), at investigations of two-phase flows in mixing layers (Yang et al., 1990). The Rayleigh – Taylor instability is able to generate the so-called "fingers" and thus – undesirable mixing of phases in the filtration ousting of one liquid by another (Yentov, 1991). Such application problems call forth the necessity to investigate first of all the general properties of the solutions of governing equations of dynamics of two-phase media. Not every solution is able to describe the real flow but only that ones which are stable to the accidental perturbations; the latter always exist in a real physical system but are not usually accounted for in the idealized mathematical statement of a problem.

2. Statement of the perturbation problem

The instability investigation for accelerating interface between homogeneous and two-phase media is performed in present study by means of small perturbations technique (Chandrasekhar, 1961). Let us consider the mechanical system, which consists of uniform incompressible liquid of density ρ_g in semi-space II and two-phase monodisperse suspension of solid particles of radius a, diluted in another incompressible liquid, in semi-space I (Fig. 2). The system has acceleration $\vec{g}$ in direction perpendicular to the interface y=0. In coordinate system XY, which is connected with the interface, both media are initially at rest and the inertia force $-\rho\vec{g}$ is acting at them.

Let us assume that at some time moment the small disturbances have arose in the system, so that the inner states in both media become non-uniform and non-stationary. The disturbed motion of the liquid is governed by the solutions of the system of equations of hydrodynamics of homogeneous medium (Landau & Lifshitz, 1986), while the two-phase mixture motion – by the solutions of the continuum model equations of disperse two-velocity medium with mutual pressure, at neglecting by the processes of mass and heat interphase exchange. Under assumption that the volume concentration of dispersed phase,

α_2, and ratio of the own (true) densities of phases ρ_1^0, ρ_2^0 are small: $\alpha_2^2 \ll 1$, $\rho_1^0/\rho_2^0 \ll 1$, the system of equations of two-phase medium motion has the form (Nigmatulin, 1990):

$$\frac{\partial \rho_1}{\partial t}+V_{1x}\frac{\partial \rho_1}{\partial x}+V_{1y}\frac{\partial \rho_1}{\partial y}+\rho_1\frac{\partial V_{1x}}{\partial x}+\rho_1\frac{\partial V_{1y}}{\partial y}=0;$$

$$\frac{\partial \rho_2}{\partial t}+V_{2x}\frac{\partial \rho_2}{\partial x}+V_{2y}\frac{\partial \rho_2}{\partial y}+\rho_2\frac{\partial V_{2x}}{\partial x}+\rho_2\frac{\partial V_{2y}}{\partial y}=0;$$

$$\rho_1\left(\frac{\partial V_{1x}}{\partial t}+V_{1x}\frac{\partial V_{1x}}{\partial x}+V_{1y}\frac{\partial V_{1x}}{\partial y}\right)=-\left(1-\frac{3}{2}\alpha_2\right)\frac{\partial p}{\partial x}-\left(1-\frac{3}{2}\alpha_2\right)nf_{\mu x}+\left(1-\frac{1}{2}\alpha_2\right)\rho_1 g_x+\left(1-\frac{3}{2}\alpha_2\right)\frac{\rho_1^0}{2\rho_2^0}\rho_2 g_x;$$

$$\rho_1\left(\frac{\partial V_{1y}}{\partial t}+V_{1x}\frac{\partial V_{1y}}{\partial x}+V_{1y}\frac{\partial V_{1y}}{\partial y}\right)=-\left(1-\frac{3}{2}\alpha_2\right)\frac{\partial p}{\partial y}-\left(1-\frac{3}{2}\alpha_2\right)nf_{\mu y}+\left(1-\frac{1}{2}\alpha_2\right)\rho_1 g_y+\left(1-\frac{3}{2}\alpha_2\right)\frac{\rho_1^0}{2\rho_2^0}\rho_2 g_y;$$

$$\rho_2\left(\frac{\partial V_{2x}}{\partial t}+V_{2x}\frac{\partial V_{2x}}{\partial x}+V_{2y}\frac{\partial V_{2x}}{\partial y}\right)=-\frac{3}{2}\alpha_2\left(1-\frac{\alpha_2}{2}\right)\frac{\partial p}{\partial x}+\left(1-\frac{3}{2}\alpha_2\right)nf_{\mu x}+\rho_2 g_x+\frac{\alpha_2}{2}\left(1-\frac{\alpha_2}{2}\right)\rho_1 g_x;$$

$$\rho_2\left(\frac{\partial V_{2y}}{\partial t}+V_{2x}\frac{\partial V_{2y}}{\partial x}+V_{2y}\frac{\partial V_{2y}}{\partial y}\right)=-\frac{3}{2}\alpha_2\left(1-\frac{\alpha_2}{2}\right)\frac{\partial p}{\partial y}+\left(1-\frac{3}{2}\alpha_2\right)nf_{\mu y}+\rho_2 g_y+\frac{\alpha_2}{2}\left(1-\frac{\alpha_2}{2}\right)\rho_1 g_y; \quad (1)$$

$$\rho_1\left(\frac{\partial e_1}{\partial t}+V_{1x}\frac{\partial e_1}{\partial x}+V_{1y}\frac{\partial e_1}{\partial y}\right)-\frac{\alpha_1 p}{\rho_1^0}\frac{d_1\rho_1^0}{dt}=\left(\left(1-\frac{3}{2}\alpha_2\right)n\vec{f}_\mu+\frac{\alpha_2}{2}\left(\rho_1\vec{g}-\nabla p\right)\right)\cdot\left(\vec{V}_1-\vec{V}_2\right);$$

$$\rho_2\left(\frac{\partial e_2}{\partial t}+V_{2x}\frac{\partial e_2}{\partial x}+V_{2y}\frac{\partial e_2}{\partial y}\right)=0; \qquad \frac{\partial n}{\partial t}+V_{2x}\frac{\partial n}{\partial x}+V_{2y}\frac{\partial n}{\partial y}=0 \ .$$

Here ρ, V_x, V_y, p, e, n are density, components of velocity vector, pressure, internal energy, number of particles per volume unit and $\vec{f}_\mu = 6\pi\mu_1 a(\vec{V}_1-\vec{V}_2)$ is Stokes force acting at individual particle. Subscript "1" denotes carrying phase, "2" – dispersive phase of particles. The first two equations express mass conservation law, from third to sixth – law of momentum conservation, the seventh – law of energy conservation for carrying phase, eighth – condition of absence of heat transfer to particles, ninth – the conservation of number density of particles.

3. Disturbances in a two-phase medium

By linearizing equations (1) near initial state of rest "0" we will have the system for the disturbed motion (Chandrasekhar, 1961). Searching for its solution in the form

$$\rho_1'=\rho_{10}\sum R_k\Psi_k\,,\ V_{1x}'=V_*\sum A_k\Psi_k\,,\ V_{2x}'=V_*\sum D_k\Psi_k\,,\ V_{1y}'=V_*\sum B_k\Psi_k\,,\ V_{2y}'=V_*\sum G_k\Psi_k\,,$$

$$p'=\rho_{10}V_*^2\sum P_k\Psi_k\,,\ S'=c_\mathrm{p}\sum C_k\Psi_k\,,\ \Psi_k=\exp(\gamma_k hy+ihx-i\omega t)\,, \quad (2)$$

we obtain the system of linear uniform equations with respect to amplitude constants. Here S – entropy, k – the disturbance type index, h – wavenumber, ω – complex frequency, V_* – some character value of velocity, asterisk denotes the disturbance of corresponding parameter. The condition of existence of non-trivial solution for this system is splitting on the two cases.

I. At $\gamma\alpha_{20}\Gamma(1-3\alpha_{20})\neq Z(\alpha_{10}\Omega_2+\alpha_{20}\Omega_1)\rho_1^0/\rho_2^0$, by equating determinant of the system to zero we obtain **the characteristic equation** in the form: $\gamma_{1,2}^2=1$. Here

$\Omega_1 = 1.5\alpha_{20}(1-0.5\alpha_{20})Z-(1-1.5\alpha_{20})F$, $\Omega_2 = (1-1.5\alpha_{20})(\rho_{20}Z/\rho_{10}-F)$, $Z = i\omega/hV_*$, $\Gamma = g/hV_*^2$, $F = \tilde{f}_0/\rho_{10}hV_*$, $\tilde{f} = 9\mu_1\alpha_{20}/2a^2$. By calculating vortex of velocity vectors of phases, we obtain: $rot\vec{V}_1' = 0$, $rot\vec{V}_2' = 0$. Thus, the *disturbances corresponding to* $k=1,2$ *are the acoustic ones, for which the phase volume densities are unchangeable*, and the *disturbed motion remains non-vortex*. We may contend that the character of these disturbances in two-phase medium remains the same as in homogeneous one, in particular, at $\alpha_2 \to 0$ they turns into the acoustic disturbances for incompressible fluid.

II. At $\gamma_3 = \left[Z(\alpha_{10}\Omega_2 + \alpha_{20}\Omega_1)\rho_1^0\right] / \left[\alpha_{20}\Gamma(1-3\alpha_{20})\rho_{20}\right]$ $(k=3)$, all the parameters are disturbed, and $rot\vec{V}_{1,2}' \neq 0$. Thus, **in a disturbed motion of two-phase medium there exist three types of disturbances, in contrary to homogeneous medium which possesses two types**.

4. Boundary-value problem for small disturbances

In domain **I** the condition for disturbances to be limited at infinity leads to requirement $\mathrm{Re}(\gamma) > 0$, which leaves two branches: $\gamma_1 = 1$, γ_3. Thus, the disturbances have the form:

$$V_{1x}' = V_*[A_{11}\exp(hy) + A_{31}\exp(\gamma_3 hy)]\Psi_0; \qquad V_{1y}' = V_*\left(B_{11}\exp(hy) + B_{31}\exp(\gamma_3 hy)\right)\Psi_0;$$
$$V_{2x}' = V_*\left(D_{11}\exp(hy) + D_{31}\exp(\gamma_3 hy)\right)\Psi_0; \qquad V_{2y}' = V_*\left(G_{11}\exp(hy) + G_{31}\exp(\gamma_3 hy)\right)\Psi_0;$$
$$P' = \rho_{10}V_*^2\Psi_0[P_{11}\exp(hy) + P_{31}\exp(\gamma_3 hy)]; \qquad \rho_1' = \rho_{10}\Psi_0\left(R_{11}\exp(hy) + R_{31}\exp(\gamma_3 hy)\right) \quad (3)$$

all the $A_{11}, D_{11}, G_{11}, P_{11}$ expressing in terms of B_{11}, and it is convenient to express $A_{31}, B_{31}, D_{31}, P_{31}, R_{31}$ in terms of G_{31}.

In domain **II** the disturbances can be written down in the following way:

$$V_{gx}' = V_* A_2 \Psi_0 \exp(-hy), \quad V_{gy}' = V_* B_2 \Psi_0 \exp(-hy), \quad p_1' = \rho_{g0} V_*^2 P_2 \Psi_0 \exp(-hy) \quad (4)$$

only that disturbance is left here which is limited at infinity. The disturbed states of the media must be conjugated at a disturbed surface of separation $y = \varepsilon(x,t)$ by means of boundary conditions, which consist in impermeability of this surface for the media, and also in equality of normal components of stresses:

$$V_{1y}' = \frac{\partial\varepsilon}{\partial t}, \quad V_{2y}' = \frac{\partial\varepsilon}{\partial t}, \quad V_g' = \frac{\partial\varepsilon}{\partial t}, \quad p' - p_g' + \sigma\frac{\partial^2\varepsilon}{\partial x^2} + (\rho_{g0} - \rho_0)g\varepsilon = 0 \quad (5)$$

where $\rho_0 = \rho_{10} + \rho_{20}$, σ is surface tension coefficient.

5. Unstable roots

Considering the elementary disturbance of the surface in the form $\varepsilon = E_0\exp(ihx - i\omega t)$ and substituting expressions (3), (4) for the disturbed parameters in boundary conditions (5) we obtain the system of four linear uniform equations with respect to four amplitude constants B_{11}, G_{31}, B_2, E_0. The vanishing of its determinant is necessary for the existence of its non-trivial solution. This leads to the **characteristic equation with respect to dimensionless complex "frequency"** Z of the form:

$$\left[1-Z\frac{Z-\frac{\rho_1^0}{\rho_2^0}\frac{F}{\alpha_{20}}}{\left(1-\frac{3}{2}\alpha_{20}\right)\Gamma}\right]\left\{\left[Z^2+Z^3\frac{Z-\left(1-\frac{3}{2}\alpha_{20}\right)\left(1+\frac{\rho_{10}}{\rho_{20}}\right)F}{\alpha_{10}\left(1-\frac{3}{2}\alpha_{20}\right)\Gamma}\right]-\left(\Sigma-\frac{\rho_{g0}}{\rho_{10}}Z^2\right)\left[1-\frac{3}{2}\alpha_{20}+Z\frac{\alpha_{10}Z-\frac{\rho_{10}}{\rho_{20}}F}{\alpha_{10}\Gamma}\right]\right\}=0$$

where $\Sigma=[(\rho_{g0}-\rho_0)\Gamma-\sigma h/V_*^2]/\rho_{g0}$. The expressions in brackets, which are enclosed in braces, at small α_{20} are close to each other and at $\alpha_{20}=0$ they are equal. Therefore the equality of the expression within braces to zero determines classic root of Rayleigh – Taylor instability:

$$(\rho_{g0}+\rho_{10})Z_{\text{hom}}^2=\rho_{10}\Sigma\,. \tag{6}$$

Accordingly to the disturbance's form (2), the instability is defined by such values of Z, for which $\text{Re}(Z)>0$. By equating of first bracket to zero, we will have the **unstable root, which is inherent only to the behavior of accelerating surface of separation between heterogeneous and homogeneous media**:

$$Z_{\text{het}}=\frac{\rho_1^0}{\rho_2^0}\frac{F}{2\alpha_{20}}-\sqrt{\left(\frac{\rho_1^0}{\rho_2^0}\right)^2\frac{F^2}{4\alpha_{20}^2}+\left(1-\frac{3}{2}\alpha_{20}\right)\Gamma} \tag{7}$$

It disappears when a dispersive phase disappears: $Z\to 0$ at $\alpha_{20}\to 0$. The absence of imagine part of the root determines **aperiodic character of the instability**, as it is for the classic root (6). The surface tension does not affect the evolution of unstable disturbances within this mechanism. This root is caused by the action of mass force since at $g\to 0$ it disappears, and action of interphase friction is the natural stabilizing mechanism for it, so, at the carrying phase viscosity increasing, $\mu_1\to\infty$, it disappears too. If two-phase mixture is located in the upper semi-plane, then, accordingly to (2), (7), the heterogeneous mechanism of instability (7) performs when the acceleration $\vec{g}$ is directed to homogeneous medium.

When surface tension is absent, the amplitude growth ratio for two mentioned roots, $\Pi=Z_{\text{het}}/Z_{\text{hom}}$, is determined by dimensionless criterion K, which itself includes the viscous kinematic coefficient of carrying phase, particle size, dimensionless wavenumber and also the ratio of the own (true) densities of phases:

$$\Pi=\sqrt{\frac{\rho_{g0}}{\rho_{g0}-\rho_0}}\left[\sqrt{K^2+\left(1-\frac{3}{2}\alpha_{20}\right)}-K\right],\quad K=\frac{9\mu_1}{4\rho_{10}a^2\sqrt{gh}}\frac{\rho_1^0}{\rho_2^0}\,.$$

When the viscosity increases or particle size decreases, or wavelength of disturbance increases, or acceleration decreases, the action of "classic" instability mechanism becomes dominant. When the parameters vary in opposite direction, the ratio of increments of the two roots goes to the finite limit: $\Pi=((1-1.5\alpha_{20})\rho_{g0}/(\rho_{g0}-\rho_0))^{0.5}$.

6. Results

As it follows from (7), the increment of amplitude growth for the heterogeneous unstable mechanism in the short waves range increases unlimitedly at $h\to\infty$. Naturally, wavelengths of disturbances must be limited from below by such values, at which they are comparable with the particle radius a, when the governing equations of motion of continuum model for two-phase medium become themselves unsuitable. Then, for the fastest disturbance, which is corresponded to this mechanism, such value of wavenumber h may be adopted, for which $ha \approx 1$. If we assume values a = 10 μm for aerosol particles, and g = 9.8 m/sec^2 – for acceleration, then we obtain the characteristic time value of e-fold amplitude growth of the fastest disturbance: $\tau \approx 10^{-2}\,\text{sec}$. This small value means the possibility

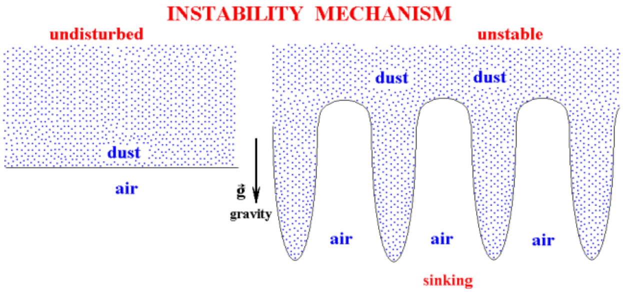

Fig. 3. The scheme of the aperiodic instability performance.

of the revealed mechanism of instability to work successfully and to lead eventually to the sinking (coagulation) of the atmospheric aerosol (Fig. 3).

If we assume $g = 10^5$ m/sec^2, which approximately equals to the value of acceleration of finest mist of daughter droplets at aerosol detonation, then we obtain $\tau \approx 3 \cdot 10^{-5}\,\text{sec}$; in this case values of increment of growth being comparable for both roots. This value is comparable with that one for evaporation time of droplets and for mixing of fuel vapors with oxidizer. The latter may mean the influence of the found hydrodynamic instability mechanism on the process of homogeneous inflammable mixture formation in two-phase detonation.

7. Conclusions

Linear instability investigation reveals new possible mechanism of aperiodic hydrodynamic instability for the case when the accelerating surface separates suspension of solid particles and homogeneous incompressible fluid. This root exists simultaneously with the classic Rayleigh–Taylor root. The found type of instability is caused by the action of inertial mass force. The found mechanism can influence the dust motion in high atmosphere leading up to the dynamic coagulation of aerosol. The results are valid for $\alpha << 1$.

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